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Motion of a Rigid Cylinder within a Shell-Core Structure

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Introduction

THE response of impulsively loaded, circular, elastic shells has recently received the attention of several investigators. A wave-type solution for the membrane stress in a long, circular shell subjected to a distributed impulse over one-half the shell circumference was derived in Ref. 1. For the same loading and shell configuration, a modal solution which includes both membrane and bending effects was obtained in Ref. 2. In Ref. 3, a solution for the axisymmetric, transient response of a long, circular shell containing an annular elastic core was presented.

In many applications, the response of elements within the shell structure must also be determined. This analysis is concerned with estimating the acceleration of a rigid cylinder within a shell-core structure when the shell is loaded by a pressure pulse distributed over one-half the shell circumference (see Fig. 1). The analysis considers a model in which the shell and core are treated as elastic; however, inertial effects in the core are neglected. Elasticity theory is used to obtain the spring effect of the core. Although wave propagation effects in the core are neglected, this simplification is supported by the fact that many core materials of interest, such as polyurethane foam, have low densities compared to the densities of other system elements.

Formulation

For a state of plane strain and neglecting shear deformation and rotatory inertia, the equations of motion, from Ref. 4, for the circular shell shown in Fig. 1 are

$$\frac{Kh^2}{12b^2}\left(\frac{\partial^4 w}{\partial\theta^4} + 2\frac{\partial^2 w}{\partial\theta^2} + w\right) + K\left(\frac{\partial v}{\partial\theta} + w\right) = -b^2\left[\rho h\frac{\partial^2 w}{\partial t^2} + \sigma_r(b,\theta) + q\right] \quad (1a)$$

$$K[(\partial^2 v/\partial\theta^2) + (\partial w/\partial\theta)] = b^2[\rho h(\partial^2 v/\partial t^2) + \tau_{r\theta}(b,\theta)] \quad (1b)$$

where w and v are the radial and tangential shell displacements; t is time; $\sigma_r(b,\theta)$ and $\tau_{r\theta}(b,\theta)$ are the core radial and shear stresses at the shell-core interface; $K = E_s h/(1 - \nu_s^2)$; ρ, h, b, E_s, and ν_s are the shell density, thickness, radius, Young's modulus, and Poisson's ratio, respectively; and the surface pressure q is given by

$$q = \begin{cases} p(t)\cos\theta, & |\theta| < \pi/2 \\ 0, & \pi/2 < |\theta| < \pi \end{cases} \quad (2)$$

The equation of motion for the rigid cylinder with density γ and radius a is

$$\pi a^2\gamma\frac{d^2x}{dt^2} = \int_0^{2\pi}[\tau_{r\theta}(a,\theta)\sin\theta - \sigma_r(a,\theta)\cos\theta]a\,d\theta \quad (3)$$

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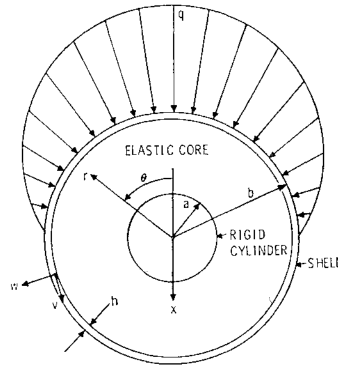

Fig. 1 Geometry of the problem.

where x is the displacement of the center of the rigid cylinder.

Solution

The applied pressure q, shell radial and tangential displacements w and v, and core stress components σ_r and $\tau_{r\theta}$ are expressed in series form as

$$q = p(t)\sum_{n=0}^{\infty} q_n\cos n\theta \quad (4a)$$

$$w = \sum_{n=0}^{\infty} w_n(t)\cos n\theta,\ v = \sum_{n=1}^{\infty} v_n(t)\sin n\theta \quad (4b)$$

$$\sigma_r = \sum_{n=0}^{\infty} S_n(r)\cos n\theta,\ \tau_{r\theta} = \sum_{n=1}^{\infty} T_n(r)\sin n\theta \quad (4c)$$

where r is the radial coordinate. The substitution of Eqs. (4) into Eqs. (1) and (3) results in a system of equations for each value of n. However, because of the orthogonality properties of the sine and cosine functions, only the system corresponding to $n = 1$ enters into the solution for the rigid cylinder displacement x. The equations for $n = 1$ are

$$K[v_1 + w_1] = -b^2[\rho h(d^2w_1/dt^2) + S_1(b) + q_1 p(t)] \quad (5a)$$

$$K[v_1 + w_1] = -b^2[\rho h(d^2v_1/dt^2) + T_1(b)] \quad (5b)$$

$$a\gamma(d^2x/dt^2) = T_1(a) - S_1(a) \quad (5c)$$

It should be noted that Eqs. (5) are identical to those that are obtained if the membrane theory of Ref. 1 is used to describe the shell response instead of Eqs. (1). In Eqs. (5), the coefficient q_1 is $\frac{1}{2}$. $S_1(r)$ and $T_1(r)$ are obtained by solving the $n = 1$ mode of the plane strain elasticity equations for the massless core, with continuity of displacements maintained at the shell-core ($r = b$) and core-rigid cylinder ($r = a$) interfaces. Then, in Eqs. (5),

$$S_1(b) = (G/b)(\sigma_1 w_1 + \sigma_2 v_1 + \sigma_3 x) \quad (6a)$$

$$T_1(b) = (G/b)(\tau_1 w_1 + \tau_2 v_1 + \tau_3 x) \quad (6b)$$

$$T_1(a) - S_1(a) = (G/a)(\delta_1 w_1 + \delta_2 v_1 + \delta_3 x) \quad (6c)$$

where

$$\sigma_1 = \frac{2}{\Gamma}\left[\frac{-2(3-2\nu)}{(3-4\nu)} + \frac{(4\nu^2 - 11\nu + 8)}{(3-4\nu)}k^{-2} - \frac{(4\nu^2 - 7\nu + 2)}{(3-4\nu)}k^2 - k^{-2}\ln k - (3-4\nu)k^2\ln k\right] \quad (6d)$$

$$\sigma_2 = \tau_1 = \frac{2}{\Gamma}\left[\frac{-2}{(3-4\nu)} - \frac{(4\nu^2 - 7\nu + 2)}{(3-4\nu)}k^{-2} + \frac{(4\nu^2 - 7\nu + 4)}{(3-4\nu)}k^2 - k^{-2}\ln k - (3-4\nu)k^2\ln k\right] \quad (6e)$$

$$\tau_2 = \frac{2}{\Gamma}\left[\frac{2(1-2\nu)}{(3-4\nu)} - \nu k^{-2} - \frac{(4\nu^2 - 7\nu + 2)}{(3-4\nu)}k^2 - k^{-2}\ln k - (3-4\nu)k^2\ln k\right] \quad (6f)$$

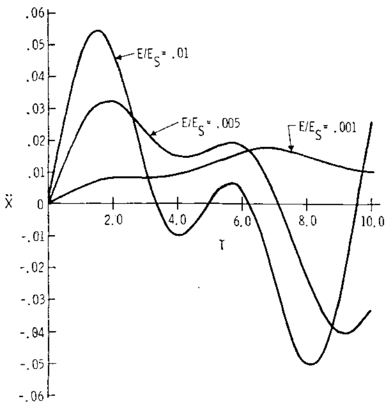

Fig. 2 Rigid cylinder acceleration.

$$\delta_1 = [4(1-\nu)/(3-4\nu)\Gamma][2-(5-4\nu)k^{-2} + (3-4\nu)k^2] \quad (6g)$$

$$\delta_2 = [4(1-\nu)/(3-4\nu)\Gamma] \times [2+(1-4\nu)k^{-2}-(3-4\nu)k^2] \quad (6h)$$

$$\sigma_3 = \sigma_1 - \sigma_2,\ \tau_3 = \tau_1 - \tau_2,\ \delta_3 = \delta_1 - \delta_2 \quad (6i)$$

$$k = a/b,\ \Gamma = (1-k^{-2})\{(3-4\nu)(1+k^2)\ln k + [1/(3-4\nu)](1-k^2)\} \quad (6j)$$

Equations (5) are solved for x by taking a Laplace transform over t with zero initial conditions. The transform variable and the transform of x are denoted by s and $\bar{x}$, respectively. Then the transformed solution for the rigid cylinder response is

$$\bar{x} = -bb_0(s^2+b_1)\bar{p}(s)/s^2(s^4+b_2s^2-b_3) \quad (7a)$$

where

$$b_0 = \frac{G\delta_1}{2a^2\gamma\rho hb},\ b_1 = \frac{K(\delta_1-\delta_2)-bG(\delta_2\tau_1-\delta_1\tau_2)}{\rho hb^2\delta_1} \quad (7b)$$

$$b_2 = [2K + bG(\sigma_1+\tau_2) - (\rho hb^2G\delta_3/a^2\gamma)]/\rho hb^2 \quad (7c)$$

$$b_3 = (G/a^2\gamma\rho^2h^2b^3)\{2\rho hbK\delta_3 + \rho hb^2G[\delta_3(\sigma_1+\tau_2) - \delta_1\sigma_3 - \delta_2\tau_3] + a^2\gamma[K(\tau_3-\sigma_3) - bG(\sigma_1\tau_2-\sigma_2\tau_1)]\} \quad (7d)$$

The time dependence of the load is taken as

$$p(t) = PH(t) \quad (8)$$

where P is the magnitude of the pressure and $H(t)$ is the Heaviside unit function. A Laplace transform of Eq. (8) is taken, and the result is substituted into Eq. (7a). Then Eq. (7a) is inverted by standard techniques and the rigid cylinder response x is twice differentiated with respect to time to give the acceleration

$$\ddot{x} = \frac{-bb_0P}{(m^2-n^2)}\left[\frac{(m^2+b_1)}{m^2}\cosh mt - \frac{(n^2+b_1)}{n^2}\cosh nt\right] - \frac{bb_0b_1P}{m^2n^2} \quad (9)$$

where $\pm m$, $\pm n$ are the roots of $s^4 + b_2s^2 - b_3 = 0$.

Results

The applied pressure time dependence is assumed to be a rectangular pulse of duration t_d. For this loading, the rigid cylinder acceleration $\ddot{x}$ is easily obtained from Eq. (9) by the principal of superposition. The dimensionless variables $\ddot{X} = \ddot{x}/\ddot{x}_{RB}$ and $T = ct/b$ are introduced, where

$$\ddot{x}_{RB} = bP/2(a^2\gamma + 2\rho hb),\ c = [E_s/\rho(1-\nu_s^2)]^{1/2} \quad (10)$$

$\ddot{x}_{RB}$ is the acceleration of the system of Fig. 1 if it were considered to be a rigid body. Curves of $\ddot{X}$ vs T are presented in Fig. 2 for $\nu = 0.10$, $\nu_s = 0.25$, $\gamma/\rho = 0.50$, $a/b = 0.25$, $b/h = 40$, $T_d = 0.07$, and $E/E_s = 0.01, 0.005, 0.001$. Figure 2 quantitatively demonstrates the reduction of the rigid cylinder acceleration $\ddot{X}$ with decreasing core Young's modulus E.

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Errors in Freestream Reynolds Number of Helium Wind Tunnels

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Nomenclature

A^*/A = ratio of effective throat-to-nozzle cross-sectional area
C_{Re_∞} = correction factor for freestream Reynolds number
M = Mach number
p = pressure, atm
q = dynamic pressure, atm
Re = unit Reynolds number, ft^{-1}
T = temperature, °K
μ = coefficient of viscosity, g/cm-sec

Subscripts

$t,1$ = reservoir stagnation conditions
$t,2$ = stagnation conditions behind normal shock
∞ = freestream conditions

Introduction

FOR nearly two decades, hypersonic fluid dynamic studies have been performed in wind tunnels employing helium as the flow medium. One of the primary reasons for using helium is that it liquifies at a very low temperature; hence, high Mach numbers can be generated in a wind tunnel without preheating the helium prior to expansion. Conventional-type wind tunnels employing helium at ambient supply temperature have operated at Mach numbers of approximately 10–26. For this Mach-number range, the corresponding freestream static temperature range is about 9°–1.5°K. Recently, Mach numbers of 30–70 were generated in the Langley hotshot tunnel, using helium as the test gas, in which calculated freestream static temperatures ranged from approximately 0.8°–12°K (Ref. 1). Heretofore, it has been common practice to use the empirical expression for viscosity of Keesom[2] in determining the freestream Reynolds number at these low temperature conditions (for example, see Refs. 3–5). However, a recent survey by one of the authors[6] revealed that usage of Keesom's expression for viscosity at these temperatures results in freestream Reynolds number errors up to 65%. The purpose of this Note is

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